1. The operator $\hat{\boldsymbol{n}}\cdot\mathbf{S}$ represents the spin projection along a direction $\hat{\boldsymbol{n}}$. We can express the direction as $\hat{\boldsymbol{n}} = \sin\theta(\cos\varphi\,\hat{\mathbf{x}} + \sin\varphi\hat{\mathbf{y}}) + \cos\vartheta\hat{\mathbf{z}}$. A beam of neutrons with energy E_0 and spin along the positive z-axis (i.e., an eigenstate of S_z with eigenvalue $\hbar/2$) enters a region where there is a uniform magnetic field **B**. The Hamiltonian interaction term with the magnetic field is

$$H = -\mathbf{B}\cdot\mu_n = 2\omega\hat{\boldsymbol{n}}\cdot\mathbf{S}$$

Where $\hat{\boldsymbol{n}}$ is the direction of the magnetic field and $\omega = B\mu_n/\hbar$.

   (a) Ignore the spatial degrees of freedom and find the state of the system $\psi(t)$ at any time $t > 0$. (10 pts)

   (b) Compute the expectation value of the spin **S**. (10 pts)

2. Consider a particle with spin $s = 1$

   (a) Derive the spin matrices in the basis $|1\ m\rangle$ of S^2 and S_z eigenstates. (10 pts)

   (b) Find the eigenstates of the spin component operator $\hat{\boldsymbol{n}}\cdot\mathbf{S}$ along the arbitrary direction $\hat{\boldsymbol{n}} = \sin\theta(\cos\varphi\,\hat{\mathbf{x}} + \sin\varphi\hat{\mathbf{y}}) + \cos\vartheta\hat{\mathbf{z}}$. (10 pts)

3. A particle of mass m is initially in the ground state (E_1) of an infinite 1-dim square of width L. Starting at $t = 0$, the system is subjected to a perturbation

$$H'(t) = V_0 x^2 e^{-t/\tau}$$

where V_0 and τ are constants.

   (a) Find the probability that the energy after time T is measured to be E_2 (first excited state). (10 pts)

   (b) Calculate (a) in the limit $T \to \infty$. (5 pts)

4. (a) Consider a particle of mass m and charge q in a one-dimensional harmonic oscillator potential $m\omega^2x^2/2$, placed in a constant electric field $\mathcal{E}$ pointing in the $\boldsymbol{x}$ direction. Derive the energy expression and the wave function for the nth excited state. (10 pts).

$$\left(a = \sqrt{\frac{m\omega}{2\hbar}}\,\boldsymbol{X} + i\sqrt{\frac{1}{2m\omega\hbar}}\,\boldsymbol{P};\ a^+ = \sqrt{\frac{m\omega}{2\hbar}}\boldsymbol{X} - i\sqrt{\frac{1}{2m\omega\hbar}}\,\boldsymbol{P}\right)$$

   (b) Calculate the energy shift in the ground state and in the degenerate 1st excited state of a 2-dimensional harmonic oscillator $H = (P_x^2 + P_y^2)/2m + \frac{1}{2}m\omega^2(x^2 + y^2)$ due to the perturbation $V = 2\lambda xy$, using first-order perturbation theory. (12 pts)

5. Particles of a given energy scatter on an infinitely hard sphere of radius a.

   (a) Calculate the phase shift $\delta_l\,(k)$. (7 pts)

   (b) For s-waves, find the values of energy (k) for which the partial cross section becomes maximal. (8 pts)

   (c) Consider the case of low energies ($ka << 1$), write an approximate expression for $\delta_l\,(k)$ and explain why the cross section is dominated by s-waves. (8 pts)