Classical Electrodynamics (I) PhD Qualifying Exam (4 problems)

Note: (1) This is a closed-book exam. Notes, dictionary, calculator, and cell phone are NOT allowed.
(2) No one can sit side-by-side with you.
(3) Terms and notations follow the textbook of J. D. Jackson, if not mentioned additionally.
(4) If answers are in a unit system different from the SI, please describe it explicitly in detail.

1. Electrostatics

(a) An infinitely long wire with a uniform linear charge density λ is located on the z-axis. Use the cylindrical coordinates (ρ, ϕ, z) to write down the electric field $\boldsymbol{E}$ in all space. (10%)

(b) The three-dimensional region with $z \leq 0$ is filled with a grounded, ideal conductor. A point charge q is located on the z-axis at $z = d$ with $d > 0$. Write down the electric potential Φ in all space. (10%)

(c) The center of a grounded, ideally conducting sphere with a radius a is located at the origin. A point charge q is located on the z-axis at $z = 3a$. Write down Φ in all space. (10%)

2. Electric dipole and dielectrics

(a) On the z-axis a point charge q is located at $z = d/2$ and another point charge $-q$ at $z = -d/2$. Consider the limiting case with $d \rightarrow 0$ to the first order of d and write down the potential Φ in all space. (10%)

(b) The three-dimensional space with $z \leq 0$ is filled with a linear dielectric with an electric permittivity ϵ. An external uniform electric field $\boldsymbol{E} = E_0 \boldsymbol{z}$ is applied in the region of $z > 0$, with $\boldsymbol{z}$ as a unit vector along the z-axis. Write down $\boldsymbol{E}$ in all space. (10%)

3. Magnetostatics

An infinitely long wire with an infinitesimal diameter of $a \rightarrow 0$ is located on the z-axis and carries a current of I which flows in the negative z direction.

(a) Write down the current density $\boldsymbol{J}$ which has a dimension of current/area. (5%)

(b) Write down the magnetic field $\boldsymbol{B}$ in all space in cylindrical coordinates. (10%)

(c) A second wire with a total length of L and an infinitesimal diameter is placed parallel to the first wire with a distance of d. Write down the force on this second wire in case that a current of I is flowing on it in the positive z direction. (5%)

4. Maxwell equations and gauge transformations

(a) Write down the Maxwell equations in vacuum with charge and current densities in integral form. (5%)

(b) Write down the Maxwell equations in media in integral form. (5%)

(c) Write down the condition of the vector potential $\mathbf{A}$ for the Coulomb gauge. (5%)

(d) Write down the condition for the Lorenz gauge. (5%)

(e) Following the problem 4d, write down the wave equations for $\mathbf{A}$ and the scalar potential Φ. (10%)