Quantum Mechanics (II) 2016 Qualify Exam

1. (a) Please write down the Klein-Gordon and the Dirac equations for a free particle with mass m.(10%)
   (b) Consider a spin ½ system represented by the normalized state vector $\begin{pmatrix} \cos \\ sin\alpha\ e^{i\beta} \end{pmatrix}$. What is the probability that a measurement of S_y yields -ħ/2?(10%)

2. Please calculate the differential cross section dσ/dΩ and total cross section σ for the Gaussian potential $V(r) = V_0 \exp(-r^2/r_0^2)$.(20 pts)

3. (a) Consider a 1-dim harmonic oscillator in its ground state $|0\rangle$ of the unperturbed Hamiltonian at $t = -\infty$. Let a perturbation $H_1(t) = -eEXe^{-\frac{t^2}{\tau^2}}$ (e, E and τ are constants) be applied between $t = -\infty$ and $t = \infty$. What is the probability that the oscillator will be in the state $|n\rangle$ (of the unperturbed oscillator) as $t \to \infty$?(15%)
   (b) The bottom of an infinite well is changed to have the shape $V(x) = \varepsilon \sin\frac{\pi x}{a}$ for $0 \le x \le a$. Calculate the energy shifts for all the excited states to first order in ε. (The well originally had $V(x) = 0$ for $0 \le x \le a$, with $V = \infty$ elsewhere) (15%)

4. (a) An electron with its spin wave function being an eigenstate of S_z with eigenvalue $\frac{\hbar}{2}$. The operator $\hat{\mathbf{e}}\cdot\mathbf{S}$ represents the spin projection along a direction $\hat{\mathbf{e}}$. We can express the direction as $\hat{\mathbf{e}} = \sin\theta(\cos\phi\,\mathbf{e}_1 + \sin\phi\,\mathbf{e}_2) + \cos\theta\,\mathbf{e}_3$. Where $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are unit vectors of the x-, y-, and z-axis, respectively. Solve the eigenvalue problem of $\hat{\mathbf{e}}\cdot\mathbf{S}$. What is the probability of finding the electron in each $\hat{\mathbf{e}}\cdot\mathbf{S}$ eigenstate? (15%)
   (b) Find the Clebsch-Gordan coefficients associated with the addition of spin ½ and orbital angular momentum l. (That is, find the coefficients α and β in $\Psi_{j,m+½} = \alpha\Psi_{lm}\chi_+ + \beta\Psi_{l,m+1}\chi_-$, with $j = l$ - ½ and $j = l$ + ½)(15%)