[1]There is a quasi-ergodic hypothesis in statistical mechanics. In short, we can set equal weight to all micro-states of an isolated system in equilibrium. This ensemble is called 'micro-canonical ensemble'. However, a system in contact with a heat reservoir, we find different weights for different micro-states depending on their energies and the temperature of the heat reservoir. This is called 'canonical ensemble' and the sum of total weights is the partition function. When a system can exchange not only energy but also particle with its surrounding, a particle-energy reservoir, we call this 'grand canonical ensemble'. Find the weight for a micro-state of a grand canonical ensemble. You should treat the system and the particle-energy reservoir as a big isolated system, apply the second law of thermodynamics and specify the parameters used in your derivation, e.g. number of particles, energy, chemical potential, temperature, number of micro-states, entropy, etc. What about the weight of a micro-state for a system which can exchange energy and volume with its surrounding in equilibrium (i.e. keeping temperature and pressure fixed)? Use k_B to denote the Boltzmann constant.

[2]A zipper with N links is used to model DNA crudely. The zipper can be unzipped only from one direction, say from top to bottom. A link can then be open only if all links above are already open. The energy of a closed link is 0 and ϵ for an open link. Determine the partition function and find the average number of open links of this DNA zipper in thermal equilibrium.

[3]Describe briefly about the behavior of free electrons in a metal. What is the meaning of Fermi energy? Write down the functional form of Fermi-Dirac distribution in statistical mechanics and determine the Fermi energy of N free electrons bounded in a metal with volume V. The Fermi energies of metals are typically at a few eV level. Comment on the contribution to the specific heat of a metal due to its free electrons at room temperature.

[4]An electron is under the influence of a magnetic field pointing along the Z direction and is in thermal equilibrium with a fixed temperature. The Hamiltonian of this system can be written down as $H = -\mu_B B \sigma_z$, where μ_B denotes the Bohr magneton i.e. the magnetic dipole moment of an electron, B denotes the magnitude of the magnetic field, and σ_z represents the z component of the Pauli spin operator. In the representation that makes σ_z diagonal, find the density matrix, ρ, and the expectation value of σ_z (i.e. $\mathrm{Tr}(\rho\sigma_z)$ for this case. What's the form of the density matrix if you choose the representation which makes σ_x diagonal? How about the expectation value of σ_z? Note that you should use $\beta(= 1/k_B T)$ in your answers.